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Interaction of Vortices in Type-II Superconductors near $T = T_c$

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The theory of Müller-Hartmann and Kramer for the interaction of widely separated vortices is extended to temperatures immediately below T_c. The interaction is found to be attractive below T_c for type-II materials with $\kappa \approx 1/\sqrt{2}$ and not too small a mean free path, in agreement with experimental observations on such materials. The phenomenon of field reversal and the work of Eilenberger and Büttner are shown to be unrelated to an attractive interaction between vortices and the consequent first-order transition at the lower critical field, at least for temperatures near T_c; it is shown how field reversal, if it occurs in the mixed state, might result in an attractive interaction at lower temperatures.

I. INTRODUCTION

In the Ginzburg-Landau theory, the interaction between vortices, at the applied field at which flux penetrates in the form of an isolated vortex, has been shown to be repulsive[1,2] for $\kappa > 1/\sqrt{2}$ and attractive[2] for $\kappa < 1/\sqrt{2}$. Since the critical value of κ for type-II superconductivity is $1/\sqrt{2}$ in the Ginzburg-Landau theory, the interaction is repulsive for type-II superconductors. This is not conclusive evidence that the phase transition at the field of first flux penetration is of second order, but it is highly suggestive; a proof that the first flux penetration is in the form of a singly quantized isolated vortex requires a demonstration that the free energy for each and every possible form of flux penetration be greater than that of a singly quantized isolated vortex.

The Ginzburg-Landau theory is, however, strictly valid only at $T = T_c$, and the nature of the initial flux penetration at lower temperatures is an open question. Recently, both theoretical and experimental evidence has been obtained for a first-order transition at the field of first flux penetration, and hence for some materials ($\kappa \approx 1/\sqrt{2}$ and mean free path l not much smaller than ξ_0) the first flux penetration is not in the form of a singly quantized isolated vortex. The experimental evidence has been partially reviewed in a previous article.[3] The most direct evidence is the observation of a first-order transition by Krägeloh, Kumpf, and Seeger[4]; this work is unpublished at the time of writing, and Krägeloh[5] and Seeger[6] merely state that the first-order transition has been observed, without giving any details. We also mention here related work on the intermediate mixed state.[7,8] On the theoretical side, the author[3,9] has proved that the initial flux penetration is not in the form of a singly quantized isolated vortex for materials with $\kappa \approx 1/\sqrt{2}$ and not too small l, and has conjectured that the initial flux penetration is in the form of a lattice of singly quantized vortices with finite spacing for these materials. If the interaction of widely separated vortices were attractive, one would have strong support for this conjecture.

In this article, we examine the interaction of vortices at temperatures less than T_c to see if it can be attractive. In Sec. II, we extend Kramer's theory[2] to temperatures less than T_c and express the interaction term in the free energy as a line integral along the boundary of a unit cell of the vortex lattice. The evaluation of this integral requires a knowledge of the behavior of the order parameter and the superfluid velocity of an isolated vortex far from the axis of the vortex; the asymptotic behavior of these quantities is determined in Sec. III and in Sec. IV we determine the values of κ and l for which the interaction is attractive. In agreement with the conjecture referred to above, we find an attractive interaction near $T = T_c$ for type-II materials with $\kappa \approx 1/\sqrt{2}$ and not too small a mean free path. In Sec. V we examine the phenomenon of field reversal and show, even if it occurs in the mixed state, that it does not result in an attractive interaction between vortices, at least near $T = T_c$, simply because field reversal does not occur for $\kappa \approx 1/\sqrt{2}$ near T_c. The situation is less clear at lower temperatures, and we indicate how field reversal might give an attractive interaction. We also consider the work of Eilenberger and Büttner,[10] and show that their results are unrelated to an attractive interaction near $T = T_c$.

II. INTERACTION ENERGY

In a previous article[11] the author has extended the work of Neumann and Tewordt[12] to obtain the exact first-order correction in $1 - T/T_c$ to the Ginzburg-Landau expression for the free energy of an inhomogeneous superconductor. Following Kramer's method,[2] in this expression we write the order parameter as $f + f'$ and the superfluid velocity as $\vec{v} + \vec{v}\,'$ where f' and $\vec{v}\,'$ are the perturbations to f and $\vec{v}$, the order parameter and the superfluid velocity of an isolated vortex; on keeping only terms up to first order in the perturbations, we find that the free energy ΔG of a unit cell of the vortex lattice, relative to the Meissner state, is given by

$$\Delta G = (H_c^2 \lambda^2 / 4\pi)(I_a + I_b) \;, \tag{2.1}$$

$$\begin{aligned} I_a = \int d^2r\, [h^2 - 2hh_a + \tfrac{1}{2}(1-f^2)^2 + \kappa_3^{-2}(\vec{\nabla} f)^2 + f^2 v^2] \\ + (1-t)\int d^2r\, \{\eta_c f^2(1-f^2)^2 - \eta_k(1-f^2) \\ \times[\kappa_3^{-2}(\vec{\nabla} f)^2 + f^2 v^2] + \eta_w f^2 \kappa_3^{-2}(\vec{\nabla} f)^2 + (\eta_{4d} + 3\eta_{4c}) \\ \times(\kappa_3^{-2}\nabla^2 f - f v^2)^2 + \eta_{4c}\kappa_3^{-2}[f^2 h^2 - 2(\vec{v}\times\vec{\nabla} f^2)\cdot\vec{h}]\} \;, \end{aligned} \tag{2.2}$$

$$\begin{aligned} I_b = 2\oint dl\hat{n}\cdot(\vec{h}\times\vec{v}\,' + \kappa^{-2} f'\,\vec{\nabla} f) \\ + 2\kappa^{-2}(1-t)\oint dl\hat{n}\cdot\{2\eta_{4c} f f'\,\vec{v}\times\vec{h} \\ + [-2\phi - \eta_k(1-f^2) + \eta_w f^2 + (\eta_{4d} + 3\eta_{4c}) \\ \times(1 - 3f^2)] f'\,\vec{\nabla} f - (\eta_{4d} + 3\eta_{4c}) f(1-f^2)\vec{\nabla} f' \\ + \eta_{4c}\vec{v}\,'\times[-f^2\vec{h} + \vec{v}\times\vec{\nabla} f^2]\} \;. \end{aligned} \tag{2.3}$$

The area integrals in Eq. (2.2) are over a unit cell of the vortex lattice and the line integrals in Eq. (2.3) are along the boundary of the unit cell; $\hat{n}$ is the unit outward normal to the boundary. The notation in Eqs. (2.1)–(2.3) is the same as that of Refs. 11 and 12; in particular, $t = T/T_c$ and lengths are measured in units of λ, the penetration depth. The first term in the expression for I_b was found by Kramer[2]; the second term gives the first-order correction in $1 - T/T_c$ to Kramer's result, and is new with this work.

For the case of widely separated vortices, the quantity I_a can be neglected when the applied field is equal to the field for initial flux penetration in the form of a singly quantized isolated vortex.[1,2,13] In addition, the quantities $1 - f$, $\vec{\nabla} f$, $\vec{v}$, $\vec{h}$, f', $\vec{\nabla} f'$, $\vec{v}\,'$, and $\vec{h}\,'$ are small along the boundary and, for the interesting values of κ ($\approx 1/\sqrt{2}$), are of approximately the same magnitude. We therefore keep only terms up to second order in these small quantities; Eqs. (2.1)–(2.3) then reduce to

$$\begin{aligned} \frac{4\pi\Delta G}{H_c^2\,\lambda^2} = 2\oint dl\hat{n}\cdot(\vec{h}\times\vec{v}\,' + \kappa^{-2} f'\,\vec{\nabla} f) \\ + 2\kappa^{-2}(1-t)\oint dl\hat{n}\cdot[(\eta_w - 2\phi - 2\eta_{4d} - 6\eta_{4c}) f'\,\vec{\nabla} f \\ - 2(\eta_{4d} + 3\eta_{4c})(1-f)\vec{\nabla} f' + \eta_{4c}\vec{h}\times\vec{v}\,'] \;. \end{aligned} \tag{2.4}$$

We now simplify the line integrals by using the symmetry of the lattice. If z is the number of nearest neighbors and the origin is chosen to be at the center of the vortex, we have

$$\oint dl\hat{n}\cdot\vec{Z} = 2z\int_0^Y dy\,\hat{x}\cdot\vec{Z} \;, \tag{2.5}$$

where $\vec{Z}$ stands for the integrands in Eq. (2.4); we consider only simple lattices with one vortex per unit cell. The integral on the right-hand side of Eq. (2.5) is from the point $(\frac{1}{2}D, 0)$ to the point $(\frac{1}{2}D, Y)$ as shown in Fig. 1; D is the distance between nearest neighbors. The perturbations f' and $\vec{v}\,'$ for widely separated vortices are given by[1,2]

$$f' = -\sum_k [1 - f(\vec{r} - \vec{r}_k)] \;, \tag{2.6}$$

$$\vec{v}\,' = \sum_k \vec{v}(\vec{r} - \vec{r}_k) \;, \tag{2.7}$$

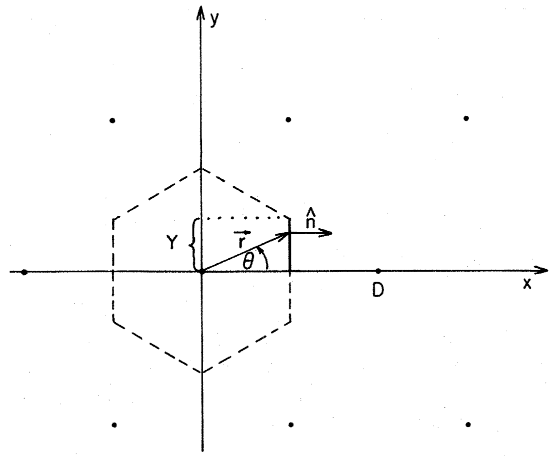

FIG. 1. Unit cell of the vortex lattice for the triangular case. The integral on the left-hand side of Eq. (2.5) is along the entire boundary of the unit cell, while the integral on the right-hand side is along the portion drawn with a solid line.

where the $\vec{r}_k$ are the position vectors of the axes of the vortices in the lattice, and the sums are over all vortices but the one at the center of the unit cell under consideration. In the following, only interactions between nearest neighbors are considered, and f' and $\vec{v}'$ may be simplified to

$$f' = f(\vec{r} - D\hat{x}) - 1 , \quad \vec{v}' = \vec{v}(\vec{r} - D\hat{x}) . \tag{2.8}$$

Since $\vec{h}(\vec{r}) = h(\vec{r})\hat{z}$ and $\hat{y}\cdot\vec{v}' = -\hat{y}\cdot\vec{v}$ along the boundary, we obtain $\hat{x}\cdot(\vec{h}\times\vec{v}') = hv\cos\theta$ where $\cos\theta = \frac{1}{2}D/r$; the term $\hat{x}\cdot\vec{\nabla}f'$ reduces to $\hat{x}\cdot\vec{\nabla}f' = -\cos\theta \times df/dr$. Finally, with $f'(\vec{r}) = f(\vec{r}) - 1$ on the boundary, we obtain

$$\frac{4\pi\Delta G}{H_c^2\ \lambda^2} = 4z\int_0^Y dy\cos\theta\Big(hv[1+\kappa^{-2}(1-t)\eta_{4c}]$$

$$+\kappa^{-2}(f-1)\frac{df}{dr}[1+(1-t)(\eta_w - 2\phi - 4\eta_{4d} - 12\eta_{4c})]\Big) . \tag{2.9}$$

The evaluation of this integral requires a knowledge of the behavior of f and v at large distances; the derivation is given in Sec. III.

III. ASYMPTOTIC BEHAVIOR OF f AND v

The quantities f and v for an isolated vortex are determined by the differential equations[11,12]

$$\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \kappa^2 f(f^2 - 1 + v^2) = \kappa^2(1-t)F(r) , \tag{3.1}$$

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{1}{r^2}v - f^2 v = (1-t)G(r) , \tag{3.2}$$

where $F(r)$ and $G(r)$ are functions of f and v first given by Neumann and Tewordt[12]; Eqs. (3.1) and (3.2), the Neumann-Tewordt equations for the order parameter and superfluid velocity, are the Ginzburg-Landau equations altered by the inclusion of the first-order correction (in $1 - T/T_c$). With $g = 1 - f$, $f_r = df/dr$, and

$$\eta_f = 2\phi + 2\eta_c + 2\eta_{4d} + 6\eta_{4c} - \eta_w , \tag{3.3}$$

Eqs. (3.1) and (3.2) can be rewritten as

$$\frac{d^2g}{dr^2} + \frac{1}{r}\frac{dg}{dr} - 2\kappa^2 g[1 + (1-t)\eta_f] = -M(r) , \tag{3.4}$$

$$\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v^2}{r^2} - v[1 - (1-t)\kappa^{-2}\eta_{4c}] = -N(r) , \tag{3.5}$$

$$M(r) = \kappa^2[g^2(f+2) + fv^2 + (1-t)U(r)] , \tag{3.6}$$

$$N(r) = (1-f^2)v - (1-t)V(r) , \tag{3.7}$$

$$U(r) = 2\phi[g^2(f+2) + fv^2] + (\eta_c + \eta_{4d} + 3\eta_{4c})[g^2(3f^3 + 6f^2 + 5f + 4)] - \eta_w[g^2(f^3 + 2f^2 + 2f + 2) + f(\kappa^{-2}f_r^2 + v^2 f^2)]$$
$$+ (\eta_{4d} + 3\eta_{4c})[6\kappa^{-2}ff_r^2 + 2f^3v^2] + \eta_{4c}\kappa^{-2}[2f^3v^2 + 3fh^2] + \eta_k[-f(1-f^2)^2 + f(-\kappa^{-2}f_r^2 + v^2f^2)] , \tag{3.8}$$

$$V(r) = (2\eta_{4d} + 6\eta_{4c} - \eta_k)[f^2(1-f^2)v] + \eta_{4c}[-\kappa^{-2}(f^4-1)v - 2f^2(1-f^2)v + 2\kappa^{-2}ff_rh$$
$$+ 2fv(fv^2 - \kappa^{-2}r^{-1}f_r) + 2\kappa^2 f_r v(f_r - r^{-1}f)] . \tag{3.9}$$

In Eqs. (3.4) and (3.5), the right-hand sides are of second and higher order in the quantities $1-f$, f_r, v, and h. We need only the asymptotic behavior of f and v, and so we rewrite the differential equations as integral equations[1]:

$$1 - f = g = K_0(cr)\int_0^r \rho d\rho I_0(c\rho)M(\rho)$$
$$+ I_0(cr)\int_r^\infty \rho d\rho K_0(c\rho)M(\rho) , \tag{3.10}$$

$$v = p\kappa_3^{-1}K_1(dr) + K_1(dr)\int_0^r \rho\, d\rho\, I_1(d\rho)N(\rho)$$
$$+ I_1(dr)\int_r^\infty \rho\, d\rho\, K_1(d\rho)N(\rho) , \tag{3.11}$$

where p is the number of flux quanta per unit cell of the vortex lattice, and

$$c = \sqrt{2}\,\kappa[1 + (1-t)\eta_f]^{1/2} , \tag{3.12}$$

$$d = [1 - (1-t)\eta_{4c}\kappa^{-2}]^{1/2} . \tag{3.13}$$

Since we are interested in the case $T \lesssim T_c$, $\kappa \approx 1/\sqrt{2}$, we may expand c and d in powers of $(1-t)$, obtaining

$$c=\sqrt{2}\,\kappa[1+\tfrac{1}{2}(1-t)\eta_f]\,, \tag{3.14}$$

$$d=1-\tfrac{1}{2}(1-t)\eta_{4c}\kappa^{-2}\,. \tag{3.15}$$

The asymptotic behavior of f and v is easily found from Eqs. (3.10) and (3.11) to be

$$1-f=g=aK_0(cr)\,, \tag{3.16}$$

$$v=bK_1(dr)\,, \tag{3.17}$$

where

$$a=\int_0^\infty \rho\, d\rho\, I_0(c\rho)M(\rho)\,, \tag{3.18}$$

$$b=p\kappa_3^{-1}+\int_0^\infty \rho d\rho\, I_1(d\rho)N(\rho)\,. \tag{3.19}$$

Our derivation of Eq. (3.16) is correct[1,2] only for $\kappa<\sqrt{2}$ at $T=T_c$; near $T=T_c$, Eq. (3.16) is correct when

$$c<2d \tag{3.20}$$

or, according to Eqs. (3.14) and (3.15), when

$$\kappa<\sqrt{2}\,[1-(1-t)(\phi+\eta_c+\eta_{4d}+4\eta_{4c}-\tfrac{1}{2}\eta_w)]\,. \tag{3.21}$$

One can easily show that this restriction on the value of κ for the validity of Eq. (3.16) does not affect our conclusions. Equation (3.17), on the other hand, is correct for all values of κ.

From Eqs. (3.16) and (3.17) one finds

$$\frac{df}{dr}=ac\,K_1(cr)\,, \tag{3.22}$$

$$h=bd\,K_0(dr)\,. \tag{3.23}$$

IV. VALUE OF κ FOR ATTRACTIVE INTERACTION

Thus armed with the knowledge of f and v at large distances, we find, on using Eqs. (3.16), (3.17), (3.22), and (3.23) in Eq. (2.9), that ΔG is given by

$$\frac{4\pi\Delta G}{H_c^2\lambda^2}=4z\int_0^Y dy\cos\theta \times\{b^2dK_0(dr)K_1(dr)[1+\kappa^{-2}(1-t)\eta_{4c}] - a^2cK_0(cr)K_1(cr)\kappa^{-2}[1+(1-t) \times(\eta_w-2\phi-4\eta_{4d}-12\eta_{4c})]\}\,. \tag{4.1}$$

We remark again that Eq. (4.1) is valid only at the field of flux penetration in the form of an isolated vortex; this field depends on p, the number of flux quanta carried by each vortex. Of the quantities a, b, c, and d, only a and b depend on p.

In this article we use Eq. (4.1) only to determine if the interaction can be attractive. For $T\approx T_c$ and $\kappa\approx 1/\sqrt{2}$, the hv term dominates at large distances when c is greater than d and the interaction is repulsive; the $(f-1)f_r$ term dominates when c is less than d and the interaction is attractive. The condition for an attractive interaction is then $c<d$, or

$$\sqrt{2}\,\kappa[1+\tfrac{1}{2}(1-t)\eta_f]<1-\tfrac{1}{2}(1-t)\eta_{4c}\kappa^{-2}\,, \tag{4.2}$$

or

$$\kappa<\kappa_{c6}\,, \tag{4.3}$$

where κ_{c6} is a critical value of κ defined by

$$\kappa_{c6}=(1/\sqrt{2})\,[1+(1-t)(-\phi-\eta_c-\eta_{4d}-4\eta_{4c}+\tfrac{1}{2}\eta_w)]\,; \tag{4.4}$$

κ_{c6} is independent of the fluxoid lattice and of p, the number of flux quanta per vortex.

To analyze our result for κ_{c6}, we extrapolate to $T=0\,^\circ$K and compare, in Table I, κ_{c6} with the values of κ_{c1}, κ_{c2}, κ_{c3}, κ_{c4}, and κ_{c5} calculated previously,[3,9] and extrapolated to $T=0$ in a similar fashion; κ_{c1}, κ_{c2}, κ_{c3}, κ_{c4}, and κ_{c5} are the values of κ for which $\kappa_1=1/\sqrt{2}$, $\kappa_2=1/\sqrt{2}$, $H_{c1}=H_c$ for initial flux penetration in the form of an isolated vortex with $p=1$, $H_{c1}=H_c$ for initial flux penetration in the form of an isolated vortex with $p=2$, and $\sigma_{NS}=0$. The differences between the κ_{ci} for $i=1$–5 have been analyzed previously[3,9]; it was concluded that the critical value of κ for type-II superconductivity was κ_{c1} for $\alpha\lesssim 50$ and κ_{c3} for $\alpha\gtrsim 50$. Another result of this analysis was the prediction of a first-order transition at the lower critical field for materials with $\kappa\approx 1/\sqrt{2}$ and not too small a mean free path l; it was conjectured that the initial flux penetration was in the form of a lattice of singly quantized vortices with finite spacing. Table I shows that the inequalities $\kappa_{c6}>\kappa_{c3}>\kappa_{c1}$ hold for $\alpha\lesssim 50$, and thus provides strong support for the conjecture. If we had found instead that $\kappa_{c3}>\kappa_{c6}$, there would exist type-II materials for which the initial flux penetration was not in the form of a singly quantized isolated vortex and the interaction between widely spaced vortices was repulsive.

Because of the approximations used in the derivation of Eq. (4.1), we can at present say very little about the form of the initial flux penetration; we

TABLE I. Values of κ_c extrapolated to $T=0\,^\circ$K as functions of $\alpha=0.882\,\xi_0/l$; κ_{c1}–κ_{c6} are defined in the text.

α	κ_{c1}	κ_{c2}	κ_{c3}	κ_{c4}	κ_{c5}	κ_{c6}
0	0.419	···	0.773	0.750	0.688	1.382
0.2	0.456	0.062	0.746	0.728	0.677	1.247
0.5	0.494	0.185	0.721	0.706	0.667	1.113
1.0	0.531	0.306	0.697	0.686	0.657	0.982
2	0.568	0.424	0.673	0.666	0.648	0.854
4	0.595	0.515	0.654	0.650	0.640	0.756
10	0.615	0.584	0.637	0.636	0.632	0.676
20	0.621	0.610	0.630	0.629	0.627	0.644
50	0.624	0.625	0.623	0.623	0.623	0.622
100	0.624	0.629	0.620	0.621	0.621	0.614
∞	0.623	0.633	0.616	0.617	0.618	0.604

have shown here only that the interaction between widely separated vortices with arbitrary p is attractive at the applied field for which the Meissner state is unstable with respect to flux penetration in the form of an isolated vortex, for materials with $\kappa \approx 1/\sqrt{2}$ and $\xi_0/l \lesssim 50$. It is likely that the initial flux penetration is in the form conjectured in Refs. 3 and 9, but we have not proven this to be the case. It is hoped to extend the above work to determine the optimum value of p and the most favorable lattice, and to calculate the applied field at which flux penetration takes place and the magnitude of the drop in the absolute value of the magnetization at the first-order transition. The simplest way to determine the form of the first flux penetration is, however, probably by means of experiment.

V. FIELD REVERSAL AND THE WORK OF EILENBERGER AND BÜTTNER

When nonlocal effects are taken into account, the penetration of a magnetic field into a superconductor is no longer governed by an exponential decay law. For materials with sufficiently small values of κ, the nonlocal effects lead to a change in the sign of the field at large distances into the superconductor; this phenomenon is called field reversal.[14] A detailed study of the effect, using the BCS kernel instead of the approximate Pippard kernel, has recently been made by Halbritter[15]; we give, in the next paragraph, a brief summary of the results which are of most interest here.

The vector potential $A(z)$, where z is the coordinate normal to the metal-vacuum interface at $z=0$, is given, in the case of specular reflection, by

$$A(z)=C\int_{-\infty}^{\infty} dk\,\frac{e^{ikz}}{k^2+K(k)}\,, \tag{5.1}$$

where C is a constant and $K(k)$ is the BCS kernel. The integral can be put into a more convenient form by means of a contour integration.[15] The singularities of the integrand are poles from the complex zeros of the denominator and a branch cut arising from the arctangent function in the kernel; the contributions are $A_1(z)$ and $A_2(z)$, respectively, so that

$$A(z)=A_1(z)+A_2(z)\ . \tag{5.2}$$

The contribution $A_2(z)$ is always negative and decreases in magnitude in an approximately exponential fashion as z goes to infinity; the decay length is approximately Ξ, where

$$\frac{1}{\Xi}=\frac{\hbar v_F}{2(\Delta^2+\pi^2 k_B{}^2 T^2)^{1/2}}+\frac{1}{l}\ . \tag{5.3}$$

The nature of $A_1(z)$ depends on the value of Ξ/λ where λ is the penetration depth of the local theory. For Ξ/λ less than about unity, there is one pole on the imaginary axis in the upper half plane and $A_1(z)$ decays exponentially with a decay length between $\lambda(\Xi/\lambda \ll 1)$ and $\Xi(\Xi/\lambda \approx 1)$. For Ξ/λ greater than about unity, there are two poles symmetrically located about the imaginary axis and the contribution $A_1(z)$ in this case displays damped oscillations which are, however, very small; these oscillations are, moreover, swamped at large distances by the contribution from the branch cut. The net effect of these two contributions is that the magnetic field for large z is negative for Ξ/λ greater than about unity and positive for Ξ/λ less than about unity.

The phenomenon of field reversal may occur in the mixed state of type-II superconductors as well, although the author knows of no evidence for this; if the effect exists here, it might lead to an attractive interaction between vortices in the following way. Close to the axis of the vortex, h and v are both positive, whereas far from the axis h and v would both be negative; in both cases, Eq. (2.9) shows that the interaction energy is positive if the hv term dominates. At intermediate (but large) values of r, however, the product hv is negative and the interaction term is negative if the hv term dominates. Equation (2.9) is, however, valid only near $T=T_c$, and one cannot be certain that field reversal, if it exists in the mixed state, can result in an attractive interaction; a theory of the interaction of vortices at arbitrary temperatures is required.

If field reversal takes place in the mixed state, the requirement for its occurrence should be much the same as in the field penetration problem, namely, $\Xi \gtrsim \lambda$. Since λ diverges at T_c, while Ξ [defined by Eq. (5.3)] remains finite as $T \to T_c$, this condition cannot be satisfied near T_c for materials with $\kappa \approx 1/\sqrt{2}$; thus field reversal has nothing to do with an attractive interaction between vortices in type-II materials near $T=T_c$: We find above an attractive interaction between vortices with a $g(=1-f)$ and a v which decreased monotonically at large distances.

It is perhaps worth pointing out that our result (3.17) for the asymptotic $v(r)$ in the isolated-vortex geometry corresponds, in the field-penetration problem, to considering nonlocal effects only insofar as they affect the temperature dependence of $K(k)$; the contribution from the branch cut is ignored. To show this, we apply the Neumann-Tewordt theory to the field-penetration problem, obtaining very easily

$$v(z)=v(0)\,e^{-dz/\lambda}\,, \tag{5.4}$$

where

$$d=[1-(1-t)\eta_{4c}\kappa^{-2}]^{1/2}\,, \tag{5.5}$$

and z is in ordinary units. If, on the other hand, we expand the kernel $K(k)$ in powers of k about $k=0$, we find

$$K(k)=\lambda^{-2}-sk^2+O(k^4)\,, \tag{5.6}$$

where

$$s=-\eta_{4c}\kappa^{-2}(1-t)\;; \tag{5.7}$$

the expansion of $K(k)$ in powers of k ignores, of course, the contribution from the branch cut. A trivial integration then yields

$$A(z)=\pi\, C\lambda\, d'\, e^{-d' z/\lambda}\;, \tag{5.8}$$

where

$$d'=(1-s)^{-1/2}=[1+(1-t)\eta_{4c}\kappa^{-2}]^{-1/2}\;. \tag{5.9}$$

The exponents d and d' in Eqs. (5.4) and (5.8) are, from Eqs. (5.5) and (5.9), identical to first order in $1-t$, provided that $|(1-t)\eta_{4c}\kappa^{-2}|<1$ so that an expansion in $(1-t)$ is valid. Since d and d' are not identical, the Neumann-Tewordt theory does not correspond exactly to expanding $K(k)$ in a power series about $k=0$. In particular, since $\eta_{4c}<0$ for all finite ξ_0/l, d diverges as $\kappa\to 0$ for fixed T less than T_c, whereas d' becomes imaginary in the same limit; both expansions are invalid at any temperature less than T_c for sufficiently small κ since the "correction" terms, the terms proportional to $1-T/T_c$, dominate as $\kappa\to 0$.

We turn now to the work of Eilenberger and Büttner.[10] As part of a program to determine the structure of a vortex at arbitrary temperatures less than T_c, they calculated the decay lengths of the order parameter and the superfluid velocity, obtaining the surprising result that these decay lengths were not real quantities for small values of κ at low temperatures; they postulated that this behavior could result in an attractive interaction between vortices. Near $T=T_c$, however, Eilenberger and Büttner found real values for the decay lengths of $g=1-f$ and v; hence their results are not related to an attractive interaction between vortices in type-II materials near $T=T_c$.

The significance of the results of Eilenberger and Büttner at low temperatures is not clear, and it is important to determine the origin of the complex decay lengths, particularly since, as Halbritter[15] has pointed out, the ranges of κ values for which (i) g and v have complex decay lengths,[10] (ii) field reversal is found in the field-penetration problem,[15] and (iii) the intermediate mixed state is observed,[5] are very close numerically; these ranges are $\kappa\lesssim 1.6$, $\kappa\leqslant 1.73$, and $\kappa\lesssim 1.7$, respectively, for pure or moderately pure superconductors. Perhaps the application of the method of Eilenberger and Büttner to the field-penetration problem would clarify the situation.

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